A theoretical analysis of the effect of altitude on running performance

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A theoretical analysis of the effect of altitude on running performance. J. Appl. Physiol. 70(1): 399-404, 1991.—A theoretical analysis of the effect of altitude on running performance is presented using a mathematical model we have recently developed and validated (19). Briefly, this model states that for any given total running duration T (in s), the average power output \( P_T \) (in W/kg) sustained by the runner is

\[
P_T = \frac{[S(1 - e^{-T/k})]/T}{\left[ \int_0^T BMR + B(1 - e^{-t/k}) dt \right]/T} \tag{1}
\]

where \( S \) represents anaerobic energy stores (in J/kg), when \( T < 420 \) s, or \( S = A - 0.233A \ln(T/420) \), when \( T > 420 \) s, \( B \) is maximal aerobic power (MAP, in W/kg) minus basal metabolic rate (BMR, in W/kg), when \( T < 420 \) s, or \( B = MAP - BMR + \epsilon \ln(T/420) \), when \( T > 420 \) s. If it is assumed that the kinetics of aerobic metabolism can be described with a single invariant time constant with no delay term, \( k \) has been set at 30 s, \( h \) has been set at 20 s as suggested by Lloyd (16), \( t \) is the time elapsed between the start \((t = 0)\) and the end \((t = T)\) of the race, and \( E \) can be considered an index of endurance capability.

For a given set of performances (e.g., the current world records for men), the corresponding composite bioenergetic characteristics \( (A, MAP, E) \) can be found using iterative approximation procedures on computer by minimizing the average absolute error between actual and estimated running times. The relationship developed by di Prampero (5, 19) between average running speed \( v \) (in m/s) and average power output \( P_v \) (in W/kg) at sea level and \( 20^\circ \)C is used in these computations

\[
P_v = BMR + 3.86v + (0.4BSA \cdot v^2)/BM + (2v^3)/D \tag{2}
\]

where \( BMR \) is \((3.5 \text{ ml } O_2 \cdot kg^{-1} \cdot \text{min}^{-1} \) or 1.2 W/kg), BSA is body surface area \( (m^2) \), BM is body mass \( (kg) \), and \( D \) is running distance \( (m) \). The second term of this equation corresponds to the nonaerodynamic cost of running, the

Athletic performance; bioenergetics; anaerobic capacity; maximal aerobic power; endurance; energy cost of running; sex differences; modeling

ELEVATION IN ALTITUDE is associated with a reduction in barometric pressure and, consequently, a reduction in air density and partial pressure of inspired oxygen (10, 12). The reduction of air density is an advantage for the runner, because it decreases aerodynamic resistance and, thus, the energy cost of running for a given speed. This is most evident at higher speeds where the aerodynamic resistance is greater. On the other hand, the reduction in maximal aerobic power with altitude is associated with a reduction in performance over distances > 400 m. The theoretical effect of altitude on running performance deteriorates. Tables of performances equivalent to the current world records at sea level on distances ranging from 60 m to the marathon, for men and women.
third term corresponds to the aerodynamic cost of running, and the fourth term corresponds to the energy spent to accelerate the body at the start of the race. Inversely, for a given set of $A$, MAP, and $k$, the average power output sustained can be estimated for any value of $T$, using Eq. 1. Accordingly, the average speed sustained and the distance covered can be computed using Eq. 2. By varying $T$ over a wide range, the performance achieved by the runner for any given distance can be found.

**EFFECT OF CHANGES IN BAROMETRIC PRESSURE**

Two factors in the proposed model of running performance are modified when altitude increases and barometric pressure ($P_B$) decreases: $I$) the maximal aerobic power and $2$) the energy spent to overcome aerodynamic resistance. The latter, which depends on air density, is the third term of Eq. 2. Anaerobic capacity (2, 6, 11, 12, 21) and endurance capability (8) do not appear to be modified by exposure to acute or chronic hypoxia corresponding to moderate altitude (less than ~4,000 m). The other terms of Eq. 2 are not affected by altitude.

The reduction in MAP with increasing altitude can be estimated from data compiled in 1981 by Cerretelli (1). We have previously shown (18) that the actual MAP, expressed in percentage of sea level MAP (%slMAP) at a given $P_B$ (in Torr), can be approximated by

$$%_{sl}MAP = a_0 + a_1P_B + a_2P_B^2 + a_3P_B^3 \quad (3)$$

with $a_0 = -174.1448622$, $a_1 = 1.0899959$, $a_2 = -1.5119 \times 10^{-3}$, and $a_3 = 0.72674 \times 10^{-6}$. This equation has been developed from data observed between 0 and 4,000 m (760 to 462 Torr). There is little relevance in studying the effect of lower $P_B$ on running performance.

The energy spent per unit of time to overcome aerodynamic resistance ($E_a$) which is given by the third term of Eq. 2, depends on air density. To take into account changes in air density associated with changes in altitude and/or temperature, the term $E_a$ can be expressed as (7, 23)

$$E_a = \left(\frac{1}{2} \cdot d \cdot AD \cdot v^3\right)/k \quad (4)$$

where $d$ is the air density (kg/m$^3$), $AD$ is the drag area, and $k$ is the mechanical efficiency assumed to be 0.25 in the equation developed by di Prampero (5). If average BSA is assumed to be 1.8 and 1.6 m$^2$ and average BM of 70 and 50 kg for men and women, respectively, the aerodynamic cost of running at sea level and $20^\circ C$ [d = 1.204 kg/m$^3$, (10)] is

$$E_a = 0.4(1.8/70)v^3 = \left(\frac{1}{2} \cdot 1.204 \cdot AD \cdot v^3\right)/k \quad (5)$$

or

$$E_a = 0.4(1.6/50)v^3 = \left(\frac{1}{2} \cdot 1.204 \cdot AD \cdot v^3\right)/k \quad (6)$$

for men and women, respectively. Accordingly $A1V/k = 17 \times 10^{-2}$ and $21 \times 10^{-2}$ for men and women, respectively. These values correspond to $AD$ of 0.30 and 0.26, which are close to those estimated by Davies (4). Therefore, Eq. 2 can be generalized for any value of $d$ and

$$PV = BMR + 3.86v + \frac{1}{2}d \cdot AD/k \cdot v^3 + (2v^3)/D \quad (7)$$

with the appropriate value for $AD/k$ according to the sex of the runner.

For a runner with known values for $A$, sea level MAP, and $E$, the actual MAP available at any altitude can be estimated with Eq. 3, taking into account changes in $P_B$. Subsequently, $P_p$ can be estimated from Eq. 1 for any value of $T$, with the actual MAP available to the runner taken into account. Finally, the average speed sustained and the distance covered can be computed using Eq. 7 with the appropriate value for $d$. The performances achieved by the runner over any given distance can, thus, be found by varying $T$ over a wide range of values.

**EQUIVALENCE OF PERFORMANCES AT VARIOUS ALTITUDES**

Tables 1 and 2 present the performances equivalent to the current sea level world records on the eight Olympic distances plus the 60-m and the 1-mile runs for men and women for selected locations at various altitudes (17). Values of $A$, sea level MAP, and $E$ have been estimated using Eqs. 1 and 2 from current world records at sea level for men (1,669 J/kg; 29.2 W/kg, corresponding to 83.8 ml O$_2$·kg$^{-1}$·min$^{-1}$; -5.62% MAP·ln $T^{-1}$) and women (1,575 J/kg; 26.4 W/kg, corresponding to 75.8 ml O$_2$·kg$^{-1}$·min$^{-1}$; -5.45% MAP·ln $T^{-1}$). The actual $P_B$ corresponding to the various altitudes has been estimated using equations describing the International Commercial Aviation Organization standard atmosphere (10) (e.g., 2,000 m, 596 Torr). A constant temperature of $20^\circ C$ is assumed. Changes in air density have been computed from change in $P_B$ using Boyle’s law ($P_B \times v = k$ e.g., 1 m$^3$ of air at 760 Torr and $20^\circ C$ becomes 1.275 m$^3$ of air at 596 Torr and $20^\circ C$; accordingly, the density is reduced from 1.204 kg/m$^3$ to 0.944 kg/m$^3$).

**DISCUSSION**

Tables 1 and 2 and Fig. 1 present, for various altitudes, the performances and the running speeds corresponding to the current sea level world record for men and women for various distances ranging from 60 m to the marathon. The projected improvement or deterioration in performances with increasing altitude should be considered as average values, because it is well known that adaptation to altitude is subject to large interindividual variations among athletes. The beneficial effect of an increase in altitude on running performance is small. Because of the comparatively slow speed sustained in running, the aerodynamic resistance is only a minor component of the energy cost. Consequently, the reduction in air density with increasing altitude cannot compensate for the associated decrease in the partial pressure of inspired oxygen and MAP. In fact, three categories of running events can be distinguished (Fig. 1).

**Short sprinting distances.** For short sprinting distances (e.g., 60, 100, and 200 m), the contribution of aerobic metabolism to the total energy production is low [<10% (19)]. Consequently the reduction in MAP with increasing altitude has little effect on the amount of
energy available to the runner. On the other hand, the average speed sustained and the aerodynamic cost of running are relatively high. The reduction of air density with increasing altitude has, thus, a modest but noticeable beneficial effect. These two factors explain why the performances on short sprinting distances steadily improve with reductions in Pa. As depicted in Fig. 1, the gain in average running speed slightly increases with the length of the race. This stems from the fact that 1) the proportion of energy spent to accelerate the body at the beginning of the race (4th term of Eq. 7) decreases while 2) the average speed sustained and the proportion of energy spent to overcome aerodynamic resistance (3rd term of Eq. 7) increase with the length of the race. Because a reduction in air density does not affect the amount of energy needed to accelerate the body at the start of the race but decreases the aerodynamic cost of running, the beneficial effect of altitude on short sprinting distances increases with the length of the race. The gain in average running speed (Fig. 1) and in performance (Tables 1 and 2) remains small, however. At moderate altitude such as Mexico City (2,240 m, d = 0.917 kg/m³ at 20°C), improvements in the current sea level world records for men would be 0.09, 0.18, and 0.45 s, respectively, for the 60, 100, and 200 m, corresponding to 1.4, 1.9, and 2.4% increases in the average running speed. As shown in Table 2 and Fig. 1, the improvements in the current sea level world records for women would be slightly higher than those observed for men (1.7, 2.2, and 2.7% increases in the average running speed for the 60, 100, and 200 m, respectively, at Mexico City). This is due to the fact that although the average speed sustained and, consequently, the aerodynamic cost of running are somewhat lower for women than for men, the energy spent to accelerate at the start of the race is lower because of the lower body mass as well as the lower speed achieved. Accordingly, a larger portion of the energy is spent in overcoming aerodynamic resistance, and the advantage of a reduction in air density is, thus, slightly greater.

The improvements in performance predicted for 60, 100, and 200 m in the present study are in reasonably good agreement with predictions made using the computation procedures suggested by Ward-Smith (23) and Frohlich (7). The procedure suggested by Ward-Smith (23) is based on the average "normalized" improvement in sprinting performances (100-400 m) observed at the 1968 Olympic Games at Mexico City and on a theoretical analysis of the energy cost of sprinting. The equation developed by Ward-Smith indicates that the increase in

### Table 1. Performances equivalent to actual sea level world records for men at selected altitudes

<table>
<thead>
<tr>
<th>Location:</th>
<th>Sea Level</th>
<th>Munich</th>
<th>Calgary</th>
<th>Albuquerque</th>
<th>Colorado Springs</th>
<th>Mexico City</th>
<th>La Paz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude, m:</td>
<td>0</td>
<td>520</td>
<td>1,045</td>
<td>1,507</td>
<td>1,823</td>
<td>2,840</td>
<td>3,658</td>
</tr>
<tr>
<td>Pa, Torr:</td>
<td>760</td>
<td>714</td>
<td>670</td>
<td>654</td>
<td>610</td>
<td>410</td>
<td>458</td>
</tr>
<tr>
<td>Air density, kg/m³</td>
<td>1.205</td>
<td>1.132</td>
<td>1.062</td>
<td>1.004</td>
<td>0.966</td>
<td>0.917</td>
<td>0.766</td>
</tr>
</tbody>
</table>

Values are given in hours:minutes:seconds.

<table>
<thead>
<tr>
<th>Distance</th>
<th>PB, Torr:</th>
<th>Altitude, m:</th>
<th>Air density, kg/m³</th>
<th>Location:</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000 m</td>
<td>14:59.65</td>
<td>3,658</td>
<td>0.733</td>
<td></td>
</tr>
<tr>
<td>10,000 m</td>
<td>1:14:40.2</td>
<td>3,658</td>
<td>0.733</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2. Performances equivalent to actual sea level world records for women at selected altitudes

<table>
<thead>
<tr>
<th>Location:</th>
<th>Sea Level</th>
<th>Munich</th>
<th>Calgary</th>
<th>Albuquerque</th>
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<th>Mexico City</th>
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<td>Air density, kg/m³</td>
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<td>0.966</td>
<td>0.917</td>
<td>0.766</td>
</tr>
</tbody>
</table>

Values are given in hours:minutes:seconds.
average speed for sprinting distances ($\Delta v$), when air density decreases from $d_0$ to $d_t$ is

$$\Delta v = \frac{\delta}{1 + 2\delta} \times \frac{d_0 - d_t}{d_0} \times V_0$$

(8)

where $V_0$ is the reference running speed and $\delta$, which is equal to 0.078, is the portion of the total power developed by the runner spent to overcome aerodynamic resistance (7.8%). According to Eq. 8, performances for 60, 100, and 200 m steadily increase with altitude. However, for a given altitude the improvement in the average speed sustained is similar for the three distances (e.g., 1.16% at the altitude of Mexico City: $= 0.917$ kg/m$^3$). This is due to the fact that the model used by Ward-Smith does not take into account that 1) a variable portion of the power output is spent to accelerate the body at the start of the race according to the length of the race and 2) the portion of the total power output spent to overcome air resistance is not constant but increases with the average speed and the length of the race. The computation procedure suggested by Frohlich (7) also neglects the energy spent to accelerate the body at the start of the race. In this procedure the power output developed by the runner is simply expressed as

$$P_v = MR \cdot v_a + \frac{1}{2} d_a \cdot AD \cdot v_a^2$$

(9)

where $M$ is the body mass in kg and $R$ is a constant (3.634 W·kg$^{-1}$·m$^{-1}$·s$^{-1}$). The value of $P_v$ is estimated from the speed sustained at sea level ($P_{v0}$). Assuming that the power output available to the sprinter is not modified by the reduction of $d$, the speed ($V_a$) sustained for a given value of $d$ ($d_a$) is the value of $V_a$ for which

$$MR \cdot v_{a0} + \frac{1}{2} d_{a0} \cdot AD \cdot v_{a0}^2 = P_{v0}$$

(10)

This procedure provides estimations of running times over 200 m, close to those computed in Table 1 (e.g., 19.37 s for men at the altitude of Mexico City, or a 2% increase in the average speed). However, the beneficial effect of altitude for performances over 60 and 100 m are probably slightly overestimated (e.g., 6.29 and 9.72 s for males at the altitude of Mexico City), because Eqs. 9 and 10 do not take into account the energy spent to accelerate the body at the start of the race.

Performances at altitude predicted by Lloyd (16) are based on a model of running performance that relates the power output developed by the runner according to the speed sustained over various race distances (from 100 to 1,500 m) to the runner’s anaerobic capacity and maximal aerobic power. This model takes into account both the reduction in air density and the reduction in maximal aerobic power with increasing altitude. The energy spent to accelerate the body is also taken into account. However, results of computations made by Lloyd (16) indicate that there is little advantage for the sprinter to run at altitude. Only a small improvement in performance is observed, with a peak occurring at the very low altitude of 1,130 m (-0.1 and -0.28 s for 100 and 200 m, respectively). Above this altitude, the detrimental effect of reduction in maximal aerobic power will negate the beneficial effect of the reduction in air density, and the performance will progressively return toward sea level performance (e.g., -0.06 and -0.24 s for 100 and 200 m, respectively, at the altitude of Mexico City). This pattern is found mainly because the model of running performance developed by Lloyd (16) does not
take into account the slow adjustment of aerobic metabolism at the start of the race (9, 13, 15, 19). In fact, this model assumes that the entire maximal aerobic power is available to the runner at the onset of the race. This leads to a large overestimation of the contribution of aerobic metabolism to the performance, mainly for sprinting distances. The detrimental effect of altitude on sprinting performance is thus overestimated.

Middle and long distances. For middle and long distances (≥800 m) the single most important factor of performance is the MAP (3, 19, 20), because the aerobic metabolism contributes 75–99% to the energy requirement of the race (19). In addition, the average speeds sustained are comparatively slow. Consequently 1) the reduction in air density with increasing altitude does not represent a major advantage while 2) the reduction in MAP markedly diminishes the power output of the runner. An elevation in altitude, accordingly, leads to a deterioration in performance, which increases 1) as P<sub>s</sub> decreases and 2) as the running distance increases, because the contribution of aerobic metabolism also increases (19) (Fig. 1, Tables 1 and 2). At the altitude of Mexico City, the reduction in the average speed sustained is ~2% for 800 m, 4% for 1,500 m, 6% for 5,000 m, and 7% for the marathon, for both men and women.

The computation procedures developed by Frohlich (7) and Ward-Smith (23) are applicable only for describing the effect of altitude on sprint performances and cannot be used on longer distances. On the other hand, estimations of performances corresponding to sea level world records have been made at various altitudes by Lloyd (16) for 800 and 1,500 m. According to Lloyd a small 1–1.5% improvement in the performances over 800 and 1,500 m should be observed at 1,130 m. This stems from the fact that Lloyd assumed that no reduction of MAP is observed at 1,130 m. Recent studies clearly indicate that a significant reduction in maximal oxygen uptake is observed at low altitude in subjects with high sea level MAP (14, 22, 24). Lloyd also underestimated the average reduction in MAP at the altitude of Mexico City (56% sea level MAP vs. 91% according to data compiled by Cerretelli (1)). The deteriorations in performances over 800 and 1,500 m at this altitude (1–2%) were, accordingly, slightly lower than those estimated in the present study.

400 m. Between the short sprint distances and the middle long distances, 400 m is run at a comparatively high speed (~90% of the speed sustained over 200 m). Thus, a reduction in air density is an advantage. However, because of the duration of the race, a sizable amount of the energy spent by the runner is provided by the aerobic metabolism (25–30%) (19). Consequently, as for middle and long distances, the reduction in MAP with increasing altitude is a disadvantage. For the composite bioenergetic characteristics corresponding to the current male world records, the advantage of the reduction in air density leads to an improvement in performance over 400 m up to an altitude of ~2,400–2,500 m, which will be the optimal altitude for the record (42.69 and 46.94 s for men and women, respectively, or 0.6– and 0.66-s improvements over the current sea level world record). Beyond this altitude the reduction of MAP is large enough to counterbalance the reduction of air resistance, and the 400-m mark slowly deteriorates. This pattern cannot be mimicked by the computation procedure developed by Frohlich (7) or Ward-Smith (23), because it is assumed that the power output available to the sprinter is not affected by altitude. On the other hand, this pattern was reported by Lloyd (16) for 400 m (as well as for 100 and 200 m; see above). However, as already mentioned, Lloyd overestimated the contribution of aerobic metabolism to the performance over short running distance, including 400 m. Accordingly, the optimal altitude estimated to better the record was comparatively low (1,130 m vs. 2,400–2,500 m in the present study).

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