Modeling human performance in running

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MORTON, R. HUGH, JOHN R. FITZ-CLARKE, AND ERIC W. BANISTER. *Modeling human performance in running.* J. Appl. Physiol. 69(3): 1171–1177, 1990.—This paper focuses on the characteristics of a model interpreting the effect of training on athletic performance. The model theory is presented both mathematically and graphically. In the model, a systematically quantified impulse of training produces dual responses: fitness and fatigue. In the absence of training, both decay exponentially with time. With repetitive training, these responses satisfy individual recurrence equations. Fitness and fatigue are combined in a simple linear difference equation to predict performance levels appropriate to the intensity of training being undertaken. Significant observed correlation of model-predicted performance with a measure of actual performance during both training and tapering provides validation of the model for athletes and nonathletes alike. This enables specific model parameters to be estimated and can be used to optimize future training regimens for any individual.

athletics modeling performance; training impulse/response

CONSIDERABLE EFFORT now is focused on the study of athletic performance. In the past, much of the focus had been anecdotal or phenomenological, but there is now an emerging body of more theoretical scientifically based research.

Banister and co-workers (2) proposed a model of the complex interaction of a number of factors contributing to athletic performance. These range from the influence of everyday life to direct intervention by training and include such intangibles as the psychological effect of good or bad execution of the performance itself in competition or even during training. Many of these factors, however, remain to be specified precisely and quantitatively. Nevertheless, some success in modeling performance has already been demonstrated in athletes (1, 3, 5) utilizing a simplified version of the original model, which only considers the input dose effect that training has on two response elements defined as fitness and fatigue (Fig. 1).

Glossary

a	Magnitude parameter in equation converting running time over a standard distance to a criterion points score that allocates 1,000 points to an existing world record, min
b	Timing parameter in above equation
Cp	Criterion performance measured on a standard task first measured as a time and then converted to a points score; it is measured regularly to assess real performance response being effected by training, points
D	Duration of training session, min
Dose	Alternative expression for quantity of training $w(t)$ absorbed in a single training session
Fitness $g(t)$	Hypothesized model component of performance ability termed fitness calculated from quantity of training undertaken, arbitrary units
Fatigue $h(t)$	Hypothesized model component of performance ability termed fatigue calculated from quantity of training $w(t)$ undertaken, arbitrary units
ΔHR	Difference between two heart rates (one usually resting heart rate)
ΔHR ratio	Ratio of elevation of exercise to maximum heart rate, with both above resting value
k_1	Arbitrary weighting factor for fitness, dimensionless (initially 1)
k_2	Arbitrary weighting factor for fatigue, dimensionless (initially 2)
L	Hypothesized upper limit of world record time, min
$p(t)$	Model-predicted performance determined from difference between weighted-model fitness $k_1g(t)$ and weighted-model fatigue $k_2h(t)$ at any time t during a training program, arbitrary units
Response	Term used in sense of pharmokinetics but applied to training, expressing some measurable result or response arising from a known input of training (dose) into a performance model
$R(t)$	World record time in a running event (1 mile in this paper), min
t_m	Time from onset of training when a maximum performance is achieved as heavy training is reduced, days
t_n	Time from onset of training (*day 0*) to day of relative poorest criterion performance, days

t_s Time from onset of training when training is reduced as peaking begins, days

w(t) Assessment of amount of training undertaken during a training session, also defined as a training impulse (trimp), or dose and calculated as the product of time (in min) spent training and ΔHR ratio, arbitrary units

Y Weighting factor applied to calculation of w(t) to increase magnitude of quantity of training nonlinearly at higher training intensities, dimensionless

τ_1 Time constant determining time course of decay in accumulated fitness g(t) between training sessions, days

τ_2 Time constant determining time course of decay in accumulated fatigue h(t) between training sessions, days

QUANTIFICATION OF TRAINING

The training dose was specified quantitatively from variables that were easily and accurately measured during training. Two such variables, which immediately suggest themselves, are the duration (D) of training and the concomitant heart rate (HR) it elicits in a trainee throughout a training session. This heart rate elevation may be regarded as an index of the fractional utilization of maximum O_2 consumption ($\dot{V}O_{2\,max}$) during even vigorous activity and may be recorded in the field by a heart rate monitor for periods ranging up to 15 h, depending on the frequency of sampling. Differences in heart rate throughout different segments of a training session, or of the steady heart rate for a whole period, may be easily distinguished and measured, as shown for example in Fig. 2.

For each exercise segment during which the heart rate is relatively constant, the product of segment duration and the concomitant fractional elevation of heart rate provides a quantitative assessment of the attendant volume of training. These products may be summed to cover the whole training bout. Thus training undertaken at time t may be quantified as an area under the curve represented by the pseudointegral

$$w(t) = (\text{duration of exercise}) \left(\frac{HR_{ex} - HR_{rest}}{HR_{max} - HR_{rest}}\right)$$

where HR_{ex} is the average heart rate during exercise, HR_{rest} is the resting heart rate, and HR_{max} is the maximal heart rate. Thus with D being the duration of exercise

$$w(t) = D(\Delta HR\ \text{ratio}) \qquad (1)$$

Furthermore, w(t) is weighted by a multiplying factor Y, which emphasizes high intensity training. The Y factor weights w(t) relative to the intensity of training undertaken during a training session as the ΔHR ratio ranges from a low to a high value (i.e., ~0.2–1.0) for a low or a high raw heart rate, respectively. The weighting factor corrects bias introduced into w(t) from inordinately long training at a low proportionate heart rate. For quantification purposes we have chosen to use $Y = e^{bx}$ in accordance with the commonly observed exponential rise in blood lactate with exercise intensity x reflected by the ΔHR ratio. Values for b were chosen to match the shape of the increment curve in blood lactate concentration (in mM) with increasing work rate and heart rate in men and women as reported by Green et al. (6). These data demonstrate that the male and female responses are sufficiently different to warrant description by separate b values for men (1.92) and women (1.67).

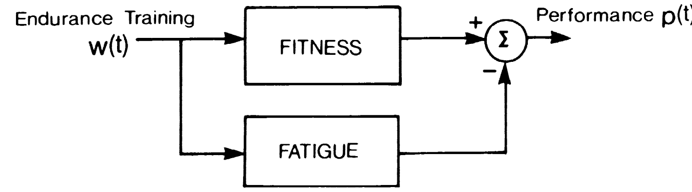

FIG. 1. Simple 2-component systems model of training and performance. Diagram shows how training input dose w(t) affects both fitness and fatigue. The summer (Σ) combines these responses, fitness positively and fatigue negatively, into a single performance output p(t).

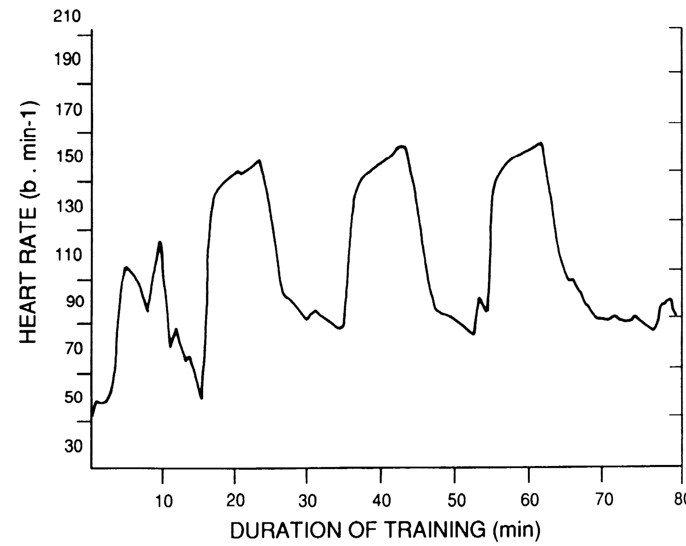

FIG. 2. Continuous heart rate record during training. During training session undertaken by this individual, several segments of different intensity are clearly identifiable from elevation of heart rate. Each segment may now be separately scored in arbitrary units of training impulse, and a precise evaluation of whole session can be obtained from sum of separate segments.

The weighting factor Y is therefore given by

$$Y = e^{bx} \qquad (2)$$

where x = ΔHR ratio. Thus w(t) becomes

$$w(t) = D(\Delta HR\ \text{ratio})\,Y \qquad (3)$$

This pseudointegral, although apparently measured in "weighted" minutes, is defined in an arbitrary unit called the training impulse or trimp and recorded for any number of sessions (or segments within sessions) of training completed each day. As an illustration of the scale, one subject (RHM) could generate ~125 training impulses by running 14 km in 1 h at a heart rate of 150 beats/min. Between training sessions, recovery from the training process takes place.

SYSTEM RESPONSE TO TRAINING IMPULSE

In the simplified model of Fig. 2, two factors, fitness $g(t)$ and fatigue $h(t)$, are recurrently affected each time training $w(t)$ is undertaken, so that

$$g(t) = g(t - i)e^{-i/\tau_1} + w(t) \tag{4}$$

and

$$h(t) = h(t - i)e^{-i/\tau_2} + w(t) \tag{5}$$

where $g(t)$ and $h(t)$ are arbitrary fitness and fatigue response levels, respectively, at the end of day t, i is the intervening period between the current day's training and that previously undertaken, and τ_1 and τ_2 are decay time constants of these respective effects.

To illustrate the behavior of the recurrent response as training is regularly repeated, if $i = 1$ and $w(t) = T$ is constant, then for fitness $g(t)$ and similarly for fatigue $h(t)$

$$\begin{aligned} g(t) &= g(t-1)e^{-1/\tau_1} + T \\ g(t) &= e^{-1/\tau_1}[g(t-2)e^{-1/\tau_1} + T] + T \cdots \\ g(t) &= T[1 + e^{-1/\tau_1} + e^{-2/\tau_1} + \cdots + e^{-(t-1)/\tau_1}] \\ &= T\frac{1 - e^{-t/\tau_1}}{1 - e^{-1/\tau_1}} \end{aligned} \tag{6}$$

$$\begin{aligned} h(t) &= T[1 + e^{-1/\tau_2} + e^{-2/\tau_2} + \cdots e^{-(t-1)/\tau_2}] \\ &= T\frac{1 - e^{-t/\tau_2}}{1 - e^{-1/\tau_2}} \end{aligned} \tag{7}$$

thus $g(t)$ and $h(t)$ rise exponentially toward respective asymptotes for a large t.

The asymptotic levels for both fitness and fatigue are determined by the training impulses score $w(t)$, the spacing of training i, and the appropriate decay time constants τ_1 and τ_2. The rate of their rise depends only on i, τ_1, and τ_2. Because fitness has a longer time constant than fatigue, it asymptotes at a higher level and a later time than fatigue. Changing the value of i while holding $w(t)$ constant does not affect the time required to reach a steady state in either response; however, the final asymptotic levels will be different. Specifically, if i is >1 the steady-state values will be reduced because of a lower volume of total training, and if i is <1 the opposite is true.

In the general case, when $w(t)$ and/or i are not constant, the above derivations must be performed numerically. The simplicity of *Eqs. 4* and *5* enables this to be done very readily on a small computer or by utilizing a pocket calculator.

To illustrate the behavior of fitness and fatigue, Fig. 3 shows the results of a uniform training regimen beginning at $t = 0$. It is assumed that $w(t) = T = 100$ trimps/day, $i = 1$, τ_1 and τ_2 are 45 and 15 days, respectively, $k_1 = 1$, and $k_2 = 2$. Fitness $k_1g(t)$ and fatigue $k_2h(t)$ (see *Eq. 8* below) are plotted together with a family of paired decay curves showing the time course of $g(t)$ and $2h(t)$ if training were to stop on any day t_s ($t_s = 10, 20, 30, \ldots, 100$). For example, with the use of Fig. 3, 60 days of uniform training would produce fitness of 3,351 arbitrary units and fatigue of 3,044 arbitrary units, with fitness exceeding fatigue by ~307 units. If training were to cease during the early stages, fatigue would continue to exceed fitness as each declined (shown by the continuous thin full lines of Fig. 3) to a decay-crossover point after which fitness dominates fatigue, and thus performance may be expected to improve. The results of beginning tapering (i.e., no training) from $t_s = 1, 10, 20, \ldots, 100$ days into a program of daily training are shown in Fig. 3. The boundaries shown inscribe the maximum separation of paired fitness and fatigue decay curves drawn from the various time points at which tapering begins. Thus if training were to cease on *day 60*, fitness and fatigue would decline, but their difference would at first increase, rising to a maximum level 1,353 units on *day 83*.

MODELING PERFORMANCE

Because fitness has a positive influence and fatigue has a negative influence on performance, model performance at time t, $p(t)$, is given by the simple linear difference

$$p(t) = k_1g(t) - k_2h(t) \tag{8}$$

where k_1 and k_2 are positive dimensionless weighting factors for fitness and fatigue, respectively.

Although k_1 and k_2 have no direct physiological interpretation, individuals with a larger k_2 may be characterized as having a fatigue-dominated performance, taking longer to recover from heavy training, whereas individuals with a higher k_1 may be described as having fitness-dominated performance, recovering quickly from heavy training during a tapering period.

To illustrate the case when $i = 1$ and $w(t) = T$ are constant, *Eqs. 6–8* yield

$$p(t) = k_1T\left[\frac{1 - e^{-t/\tau_1}}{1 - e^{-1/\tau_1}}\right] - k_2T\left[\frac{1 - e^{-t/\tau_2}}{1 - e^{-1/\tau_2}}\right] \tag{9}$$

where the denominators are each constant, determined only by the individual's respective time constants for fitness and fatigue.

Thus performance response $p(t)$ first declines from zero to a negative minimum at a time t_n given by

$$t_n = \frac{\tau_1\tau_2}{\tau_1 - \tau_2}\ln\left[\frac{\tau_1k_2(1 - e^{-1/\tau_1})}{\tau_2k_1(1 - e^{-1/\tau_2})}\right] \tag{10}$$

and thereafter increases, and for a large t becomes positive and asymptotic at

$$p(t) = \frac{k_1T}{1 - e^{-1/\tau_1}} - \frac{k_2T}{1 - e^{-1/\tau_2}} \tag{11}$$

In the general case, for varying $w(t)$ and/or i, numerical methods are easily employed.

If training ceases on day t_s, $w(t)$ becomes zero, and both fitness and fatigue decay exponentially from their accumulated levels $g(t_s)$ and $h(t_s)$, both usually from near their asymptotes. Performance is then modeled by

$$p(t) = k_1g(t_s)e^{-t/\tau_1} - k_2h(t_s)e^{-t/\tau_2} \tag{12}$$

To elucidate the behavior of predicted performance bet-

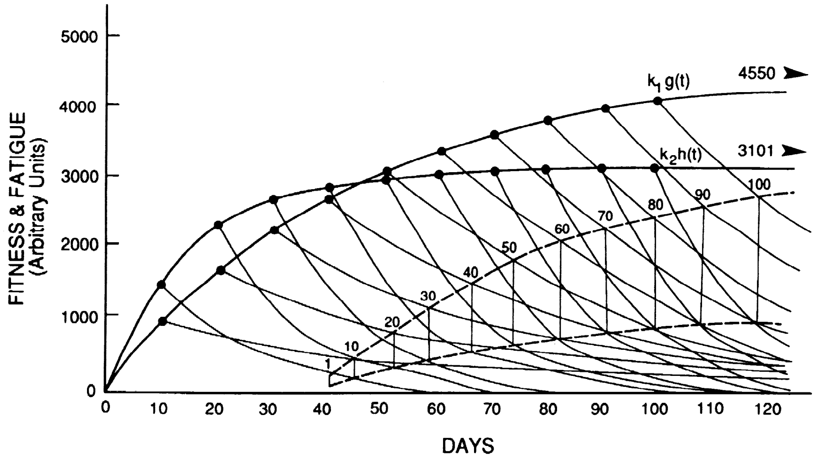

FIG. 3. Illustrative behavior of fitness and fatigue during and after uniform daily training of 100 trimps. Bold lines indicate exponential growth of fitness [$k_1g(t)$] and fatigue [$k_2h(t)$] under this training regimen with $\tau_1 = 45$, $\tau_2 = 15$, $k_1 = 1$, and $k_2 = 2$. If training is terminated at any time t_s, recovery takes place as functions decay (thin lines). Maximum positive differences between fitness and fatigue are shown by vertical bars. These maximum bars are located at days on which they actually occur after completing number of training days (t_s) heading each bar. Their lengths indicate magnitude of each performance in arbitrary units.

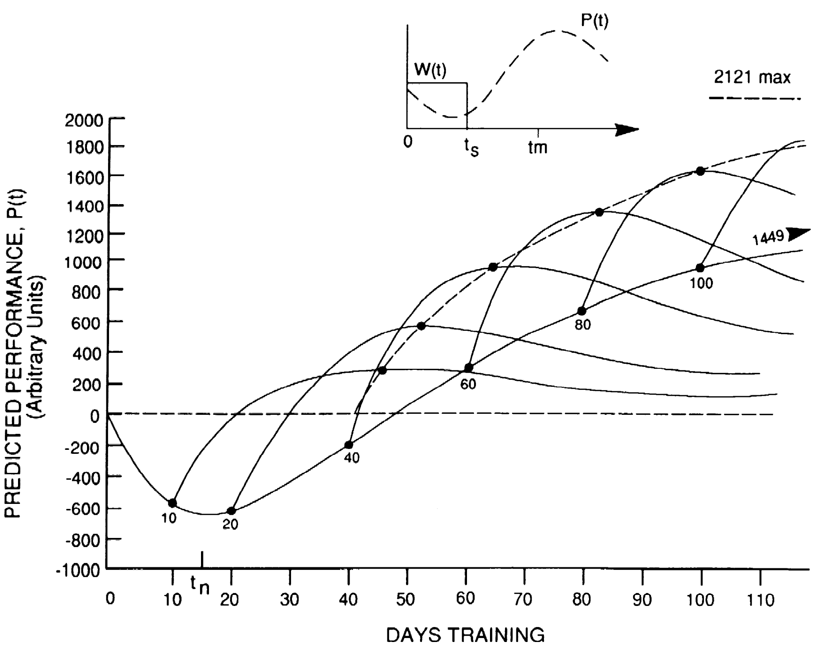

FIG. 4. Illustrative behavior of performance during and after uniform daily training of 100 trimps. Lower full curve represents performance that would result if training at current rate continued indefinitely. In this case, recovery to baseline performance takes ~47 days. For continuous daily training at 100 trimps, performance asymptotes toward a value of 1,449 as t becomes large. If instead training is terminated after an interval t_s, then performance increases (see *inset* for time course of single case) described by family of curves branching upward (t_s = 10, 20, 40, 60, 80, 100 days). Dashed line curve passes through loci of respective performance peaks at time t_m (bars of Fig. 3), asymptoting toward the eventual value of 2,121 units that would occur 16 days after end of an extended period of continuous training. A better performance could only result from a new training regimen at a higher training impulse value per day. This process could theoretically proceed up to the limit of a person's genetic potential.

ter, its time course is illustrated in Fig. 4 and is shown by the lower full line initially dropping below the zero axis, representing the level of initial performance at the onset of training.

Unremitting heavy training, although at first producing increasingly poorer predicted performances up to day $t_n = 16$ of training as fatigue dominates, allows performance to recover thereafter to the baseline level by *day 47.* Continued training without respite after this crossover point is reached is then marked by continued improvement of performance toward an asymptote of 1,449 units, provided sickness does not result from over training.

If continuous training were to cease on day $t_s = 10$, 20, 40, 60, 80, and 100, then the corresponding upward branching performance curves illustrate predicted performance during this inactive period. It is noteworthy that ceasing this level of training at $t_s = 60$ days produced a predicted maximum performance of ~1,353 units by *day 83,* shown by t_m in Fig. 4, *inset,* which is almost equivalent to that predicted to be produced asymptotically beyond *day 120* were daily training able to be continued at this level. No decay crossover point between fitness and fatigue is evident after 47 days of training irrespective of continuance or cessation of training, because past this point fitness is continually greater than fatigue.

In practice, athletes taper their training by steady reduction over a short period before competition. Complete cessation is the most dramatic form of tapering, but the general pattern of predicted performance produced by any degree of variation in tapering does not appear to differ significantly from that outlined above.

The mathematics of the model has considerable intuitive appeal and logic, and, if the model is valid, predicted performance should be related to actual timed or measured performance.

QUANTIFYING ACTUAL PERFORMANCE

To assess actual performance, trials called criterion performances should be completed and recorded. These trials have two important aspects. They must represent best-effort performances on a standard test that is appropriate in length and intensity of effort to the competition event being prepared for. Furthermore, they must be measured as frequently as possible throughout training and competitive periods. They reflect occasions when the athlete may not be expected to perform well, such as when fatigue is predominant due to the commencement of a period of particularly heavy training, as well as other occasions, such as during tapering for competition, when performance may be expected to be much better. At first these criterion performances are recorded as times for the standard distance and then they are transformed to scores on a criterion points scale.

This transformation of best-effort performance times reflects the logic that a 10-s improvement, relatively easily achieved by an 8-min miler, is a more difficult proposition for a 6-min miler and a highly significant achievement for a 4-min miler. That is, the transformation must be nonlinear, because recorded absolute time improvements are worth more points toward the asymptote of the growth curve (elite level) than in its rapidly rising early phase (representing the level of the beginning or average performer).

These ideas are embodied in the way world track records have improved exponentially over the years (8) and now show asymptotic tendencies. For example, improvement in the 1,500-m world record [R(t), in min] at time t follows the curve

$$R(t) = 3.1 + 1.065e^{-0.01t} \tag{13}$$

where the time is measured in years starting in 1896 (t = 0). This identifies the mathematical form of the chronological trend in world records as

$$y = L + ae^{-x/b}$$

where L is an ultimate limit, a is an amplitude parameter that is positive for running events and negative for throwing and jumping, and b is a time parameter. That is

$$x = b \ln \frac{a}{y - L}$$

In the application of this transformation, y is the time or distance recorded for a criterion performance and x is the associated points score. For the 1,500 m, L is identified (8) as 3.1 min, indicating an infinite points score, and a and b are determined in the following way.

A world best time of 3.5 min may be conveniently set at 1,000 points. An arbitrary assumption may also be made that any able-bodied healthy individual ought to be able to cover a distance of 1,500 m in 15 min for a zero score. Thus given a recorded criterion performance time y (in decimal form) for the 1,500 m, mathematical manipulation of an equation of the form shown above converts it to a criterion points score Cp given by the equation

$$Cp = 294.7 \ln\left[\frac{11.9}{y - 3.1}\right] \tag{14}$$

Performance times for other distances or events relevant to the ultimate effort being prepared for may be treated in exactly the same manner and used to establish an appropriate criterion performance points equation.

It is criterion points gained in this manner against which the predicted performance scores, derived as described earlier, are iteratively modeled (patterned) by changing the parameters τ_1, τ_2, k_1, and k_2 to obtain the least-squares best fit of predicted to real performance. When this is achieved, these parameters may be used for a period to prescribe both the dose and pattern of training necessary to produce desired future performance.

EXPERIMENTAL VALIDATION OF THE MODEL

The mathematical model of training elaborated above has fitted real data derived during the ongoing study of athletes (1–3, 5). Understandably, however, athletes and coaches suspect scientific interference in the art of coaching. Who among them, for instance, would believe in the illustrative case noted earlier that, relative to a normal training dose, a minimum or no training is required for ~16 days before competition to avoid a negative effect of training on competition performance?

Thus compliance with the strict requirement for collecting reliable and frequent data on training and best-effort performances throughout a training program has not been properly achieved. This has been detrimental to the accuracy of the modeling procedures.

Two of us, therefore, *RHM* and *EWB*, undertook to train at least once each day continuously for 28 days in a training experiment. Before commencing the program each subject was medically approved to participate and signed the informed consent approval required by University Ethics Rules on Human Experimentation. Before beginning training *RHM* had been only mildly active but was nevertheless quite fit, and *EWB* (a lifetime jogger) had been training by running once or twice per week for 45–50 min on each occasion. Basic physical details of the subjects are shown in Table 1.

Both subjects understood that the requirements of the program would be that they commit to training for 28 days, once per day for 40 min during the first 7 days and then twice per day for 40–50 min each time for a further 21 days. They also undertook to complete a timed run to the best of their ability over a prescribed distance, termed a criterion performance, at least twice per week. Usually this criterion performance was incorporated into training and completed at the beginning of a training bout. Criterion test results for the standard length run (4.7 km undulating for *RHM* and 4.2 km flat for *EWB*) for each subject ranged between 17 and 23 min at various stages in the training period. Training was first undertaken on a cycle ergometer and lasted 50 min on each of the first 5 days. At this point the activity was changed to running for the same period. On the 11th day two training sessions per day of 40–50 min each were started, and this regimen was continued up to the 28th day.

After the period of intense training, each subject ceased formal training for 50 days, carrying out only the physical exercise necessary to complete the exhaustive cycle ergometer tests and criterion performances.

TABLE 1. *Subject details*

Subject	Sex	Age, yr	Height, cm	Weight, kg	$\dot{V}O_{2\,max}$, $ml \cdot kg^{-1} \cdot min^{-1}$
EWB	M	57	180	86	48.8±2.1
RHM	M	42	178	73	51.3±1.7

TABLE 2. *Summary of model constants and statistics for least-squares regression of criterion performance on predicted performance*

Subject	τ_1	τ_2	k_1	k_2	r^2	*F* Statistic	df	*P*	SE
EWB	50	11	1	1.8	0.71	59	4,21	≤0.001	±13
RHM	40	11	1	2.0	0.96	252	4,18	≤0.0001	±7

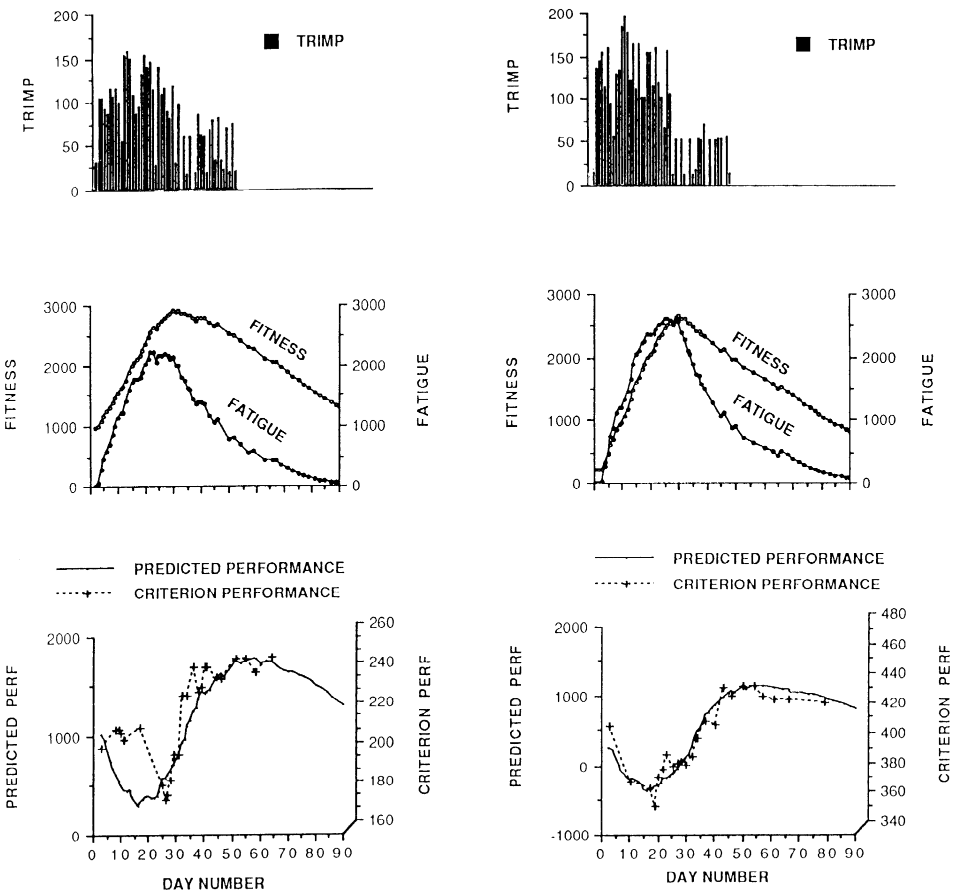

FIG. 5. Experimental results for 2 subjects, *EWB* (*left*) and *RHM* (*right*). *Top*: distribution of daily training impulse throughout training (28 days) and tapering phases of experiment. *Middle*: fitness and fatigue curves calculated from training impulse. These represent fitness and fatigue appropriate to a least-squares iterative matching of predicted to actual performance for each subject. *Bottom*: best matching of predicted and criterion performance scores from modeling process (solid and dashed lines, respectively). A good degree of fit may be observed.

During each training session and laboratory test, heart rate response to exercise stress was recorded every 15 s from electrodes on a small FM transmitter secured to the chest. This signal was received and stored in a watch worn on the wrist (Polar Electro) for later downloading and analysis. Thus two important elements quantifying the training impulse of an exercise period, i.e., the exact duration and concomitant heart rate of every session or subperiod of a session, could be assessed and a relevant training impulse $w(t)$ calculated for each training session.

Predicted performance $p(t)$ was deduced from the contribution of $w(t)$ to fitness and fatigue as outlined. Real measures of performance were converted to criterion points throughout training and detraining on a scale as described above. The patterns of predicted and real performances were iteratively modeled against each other to obtain a best fit between them along the whole time course of training and tapering. That is, given initial values for τ_1, τ_2, k_1, and k_2, the iterative procedure minimizes the sum of squared deviations

$$\sum_j [Cp(j) - p(j)]^2$$

between criterion points derived from *Eq. 14* and the model-predicted performance from *Eqs. 8, 4,* and *5* with the known input stream $w(t)$, taken over times j at which performance trials were recorded. The resulting solutions for each subject are shown in Table 2.

The coefficients of determination, r^2, giving the proportion of the sum of squares explained by the model, are highly significant for both subjects. So too are the *F* statistics, which are the ratios between the mean sum of squares explained by the best fits and the residual mean squares, with corresponding degrees of freedom, in each case. An initial fitness of 1,000 units was allocated to *EWB* to account for his moderate degree of fitness at the onset of strenuous training.

Figure 5 summarizes the experimental results for each subject: the pattern of training dosage, the fitness and fatigue responses, and the predicted and actual performance.

Besides the statistical and visual goodness of fits, several other features of these data are consistent with theoretical aspects of model behavior, particularly with respect to *RHM*, whose predicted and real performance corresponded more closely. First, t_n, calculated from model constants as 11 days, agrees quite well with the 15–19 days, which may be visually determined from Fig. 5 for real performances. Also, t_n for *EWB*, calculated as 16 days, agrees less well with reality (25–26 days), which may have been due to *EWB*'s relatively better basic condition at the onset of training. Second, it is obvious that despite continuance of heavy training, again more evident in *RHM* than in *EWB*, criterion performances began to improve from t_n toward their onset value. Theory predicts that with a constant daily training impulse, an initial decrease in criterion performance would begin to revert to its baseline (onset) level at a time depending on the respective time constants τ_1 and τ_2. Lastly, the time course of improving performance from the onset of tapering (in the present case from *day 28* onward) is extensive and much longer, in the present cases lasting almost 30 days (although this was undoubtedly provoked to some degree by the extensive testing regimen), than the time usually allowed by athletes before important competition. The optimal peaking period for elite athletes has not been reported in the literature, probably due to the fact that carefully controlled experiments such as the one reported here are not normally carried out on elite athletes. The period ($t_m - t_s$), would likely be less long for them, and its value would be embodied in the athlete's set of personal model constants.

SUMMARY

Theoretical and quantitative aspects of a simple model of the dose-response relationships between training, fitness, fatigue, and athletic performance were described. The model possesses intuitive, mathematical, and physiological elegance. Although previous studies (1–3, 5) have shown some success modeling real performance, they have suffered from being somewhat uncontrollable by the investigators and observational rather than manipulative. The practical difficulty of course is that in reality athletes and coaches are unwilling to allow unusual or drastic changes by scientists to carefully laid training plans. The results of training manipulations described here support the model theory and indicate that once individual-specific model parameters are derived, accurate prediction of the time course of performance from quantitative doses of training is possible for any healthy individual. Now that the elements of performance, fitness, and fatigue have been established, it remains to attempt to pattern them against identifiable physiological or biochemical changes induced by training, such as those that may be derived from the analysis of respiratory gas exchange or from the blood chemistry changes alluded to earlier.

This work was supported by funds from the Natural Sciences and Engineering Research Council of Canada.

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Received 17 February 1989; accepted in final form 30 April 1990.

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