Adequacy of a systems structure in the modeling of training effects on performance

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Busso, Thierry, Claude Carasso, and Jean-René Lacour. Adequacy of a systems structure in the modeling of training effects on performance. J. Appl. Physiol. 71(5): 2044-2049, 1991.—A systems model of training effects on performance was applied to eight initially untrained subjects who were volunteers for an endurance training program for the purpose of verifying the statistical adequacy of the systems structure. In the model initially proposed by T. W. Calvert, E. W. Banister, M. V. Savage, and T. Bach (IEEE Trans. Syst. Man Cybern. 6: 94-102, 1976), the performance changes were related to the successive training loads by three first-order transfer functions. In the present study, the number of first-order components was statistically tested. A model including only one component, which had a positive effect on the performance, provided a significant fit with the performances in every subject. A second component significantly improved the fit in only two subjects. This further component, which had a negative effect on performance, was identified as fatigue. Nevertheless, a two-antagonistic component model is proposed to provide a good representation of the training responses. However, the low level of exercise demands and the inaccuracy of the fit could have impaired the evidencing of a fatiguing effect during the presently studied training protocol.

mathematical modeling; systems theory; physical training; fitness; fatigue

FEW ATTEMPTS have been made to apply systems theory to the description of human responses during physical training (1–3). According to this description, the human body is represented as a system reacting to different stimuli. The set of stimuli imposed on the human body during physical training is represented as a single input, i.e., the total amount of training. The set of responses to these stimuli is represented as a single output, i.e., the performance achieved in the subject’s activity. The physiological processes responsible for these responses are considered as systems controls, relating the output to the input. The complexity of the physiological processes involved during training cannot be entirely taken into account by a systems model of training responses. Such a model is supposed to summarize the data concerning the training and performance in a general transfer function. It is more a model of data than a model of structure, even if the aim of the model is to single out the fatigue and fitness responses from the observed behavior.

Initially, Calvert et al. (3) proposed to describe the systems behavior with two antagonistic systems controls ascribed to the fitness and fatigue responses. The positive training effect (improvement of fitness) was described with a second-order transfer function, which could be considered as the difference between two first-order functions. Another first-order transfer function was thought to describe the fatigue due to the training. In subsequent studies (1, 2), the fitness response was represented by a first-order transfer function. With this two-component representation, the gain of fitness was thought to be immediate after the completion of a training session. In the two- and three-component models, the performance, i.e., systems output, was the balance between the fitness and fatigue responses.

In the above-mentioned studies (1–3), the statistical confidence of the estimates was not computed. Any correct interpretation of the system’s behavior implies that the adequacy of the model was statistically quantified (5). A sufficient statistical analysis is still lacking to test the accuracy of the proposed systems model.

For the purpose of analyzing the adequacy of the systems structure, different levels of complexity in the model formulation were tested in sedentary subjects, volunteers for an endurance training program performed on a cycloergometer.

Glossary

D Duration of exercise, min
\( g_r(t) \) Impulse response of component \( r \)
\( k_r \) Magnitude factor of component \( r \), arbitrary units (KU)

MAP External maximal aerobic power, W
\( N \) Number of measurements of performance output
\( P \) Average power output during exercise, W
pi

\( p^* \) Basic level of performance, arbitrary units (PU)
\( p_i \) Actual performance on day \( i \), arbitrary units (PU)
\( \hat{p}_i \) Estimated performance on day \( i \), arbitrary units (PU)

\textbf{PS1} Power sustained during 1 h, W

\textbf{p(t)} Time function of performance
\textbf{R} Number of systems components
\textbf{w}_i Training load on day \( i \), arbitrary units (TU)
\textbf{w(t)} Time function of training load
\textbf{\( \tau_r \)} Decay time constant of component \( r \), days

\textbf{METHODS}

\textit{Experimental methods.} Eight sedentary male subjects, mean age 20 (19–22) yr, volunteered for a 14-wk endurance training program. Their maximal oxygen uptake rate (\( \text{VO}_2 \max \)) was 51.3 (41.6–57.9) ml·min\(^{-1}\)·kg\(^{-1}\) before the study. They trained on a cycloergometer for four 1-h sessions per week. A 2-wk rest period was inserted during the 10th and 11th wk. \( \text{VO}_2 \max \) was measured every 3 wk during a progressive exercise performed until exhaustion. The external maximal aerobic power (MAP) was calculated as the product of the \( \text{VO}_2 \max \) and the net efficiency, estimated from submaximal steady states. These experimental methods were described in detail in a previous study (4). The subjects trained at 60–90% of \( \text{VO}_2 \max \) according to their subjective feeling of well-being or fatigue. The performance output of all the subjects was expressed on the same scale to avoid the differences in performance level that could distort the comparison in individual model parameters. The score of 100 arbitrary performance units (PU) was allocated to the first measured performance (day 4 or 5) in each subject (PS1\(_{\text{init}}\)). The performances measured throughout the experiment were referred to the first one: \( p(t) = 100 \text{PS1}(t)/\text{PS1}_{\text{init}} \).

Model development. The training subject is represented by a system whose input is the total training load \( w(t) \) and the output is the performance \( p(t) \), both of which are functions of time \( t \). The working of the system is considered to be described by \( R \) additive first-order systems. The present formulation of the systems model is more general than the previously proposed model (1–3). It allows the adequacy of the number of components introduced in the model to be tested.

The mathematical form of the impulse response of a first-order system is \( g(t) = ke^{-t/\tau} \). A first-order system is characterized by a decay time constant \( \tau \) and a magnitude factor \( k \). This factor is considered negative or positive for the systems control that induces, respectively, a performance decrease or increase. It is expressed in arbitrary units (KU), depending on the units used in the estimation of the systems input and output.

The performance is obtained by the convolution product (indicated by an asterisk) of the training loads \( w(t) \) with the impulse responses of each system \( g_i(t) \), with \( r \) varying from 1 to \( R \)

\[
p(t) = p^* + w(t) * g_1(t) + \cdots + w(t) * g_R(t)
\]

where \( p^* \) is the basic level of performance. This level is determined by the genetic endowment and the prior habits of the subjects, although \( p^* \) can be improved by training. It corresponds to the part of fitness that decreases sufficiently slowly to be considered constant during the period of study.

The convolution product is defined by

\[
w(t) * g_i(t) = \int_0^t g_i(t - t')w(t') \, dt'
\]

The discretization of Eq. 2 results in

\[
w(n\Delta t) * g_i(n\Delta t) = k_i \Delta t \sum_{i=0}^{n-1} w(i\Delta t)e^{-(n-i)\Delta t/\tau},
\]

where \( t = n\Delta t \) and \( w(0) \) is the initial status, resulting from the inputs preceding the studied period. The value of \( \Delta t \) is 1 day. Let \( w_i \) be the training input on day \( i \) \( w_i = w(i\Delta t) \). In the present study, the subjects were initially untrained; the initial status was supposed to be nil \( w(0) = 0 \). Therefore the performance on day \( i \) is estimated by

\[
\text{TABLE 1. Maximal aerobic power and endurance capability before and after the experiment}
\]

<table>
<thead>
<tr>
<th>Subject</th>
<th>MAP, W</th>
<th>Before</th>
<th>After</th>
<th>PS1, W</th>
<th>Before</th>
<th>After</th>
<th>Fraction of MAP, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>230</td>
<td>263</td>
<td>157</td>
<td>196</td>
<td>68.3</td>
<td>74.5</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>327</td>
<td>337</td>
<td>251</td>
<td>294</td>
<td>76.8</td>
<td>87.2</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>294</td>
<td>346</td>
<td>194</td>
<td>245</td>
<td>66.7</td>
<td>71.4</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>294</td>
<td>332</td>
<td>193</td>
<td>271</td>
<td>67.7</td>
<td>81.6</td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>294</td>
<td>327</td>
<td>203</td>
<td>272</td>
<td>81.3</td>
<td>83.2</td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>300</td>
<td>318</td>
<td>221</td>
<td>291</td>
<td>73.7</td>
<td>80.8</td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>281</td>
<td>309</td>
<td>180</td>
<td>239</td>
<td>64.1</td>
<td>77.3</td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>188</td>
<td>238</td>
<td>130</td>
<td>191</td>
<td>72.3</td>
<td>80.3</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>276</td>
<td>308</td>
<td>197</td>
<td>246</td>
<td>71.3</td>
<td>79.6</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>36</td>
<td>36</td>
<td>39</td>
<td>38</td>
<td>6.8</td>
<td>6.0</td>
<td></td>
</tr>
</tbody>
</table>

MAP, maximal aerobic power; PS1, power sustained during 1 h; fraction of MAP, fraction of maximal aerobic power sustained during 1 h.
\[ \hat{p}_n = p^* + \sum_{r=1}^{R} k_r \sum_{i=1}^{n-1} w_i e^{-(n-i)r} \] (4)

The set of model parameters is determined from the pool of actual performances by minimizing the residual sum of squares (RSS)

\[ \text{RSS}(N) = \sum_{n} (p_n - \hat{p}_n)^2 \] (5)

with \( n \) taking the \( N \) values corresponding to the day when the actual performance is measured.

If the values of the different decay time constants are fixed, RSS becomes a linear function of the \( R + 1 \) parameters, \( p^* \) and \( k_r \), where \( r = 1, \ldots, R \). These parameters can be estimated by multiple linear-regression methods. With a grid of values on the set of decay time constants (between 1 and 150 days), successive minimizations of RSS give the total set of model parameters. The computation was performed with a number of components varying from one to four.

**Statistical analysis.** The indicators of the goodness-of-fit were estimated for all the levels of model complexity, i.e., for one to four first-order components. The correlation \( r \) between the estimated and actual performances was calculated. The statistical significance of the fit was tested by an analysis of variance on the RSS. The degrees of freedom are \( 2R \) for the variations explained by the model and \( N - 2R - 1 \) for the residuals. The statistical \( F \) ratio test was used to test the level of significance of the model fit. The SE was calculated as the square root of the mean residual square, \( \text{RSS}/(N - 2R - 1) \).

The adequacy of the addition of a component was tested by an analysis of variance on the RSS relating to this change. The decrease of the residual variations explained by the introduction of a further component has a degree of freedom of 2. The degree of freedom of the residuals is reduced by 2. The statistical \( F \) ratio test is used to test the level of significance of the added component. If the addition of one component did not significantly improve the explanation of the performance variations, the addition of two components was tested with the corresponding degrees of freedom.

The paired \( t \) test was used to examine the statistical significance of the changes in MAP, PS1, and the fraction of MAP sustained during 1 h, occurring during the experiment.

**RESULTS**

The training program provided a gain of fitness; there was a significant increase in the MAP (+12%, \( P < 0.001 \)) and in the fraction of MAP the subjects could sustain for 1 h (+12%, \( P < 0.001 \)). Thus the performance level (PS1) increased significantly during the experiment (+24%, \( P < 0.001 \)). Table 1 shows the individual variations of these parameters.

The systems model, with one first-order component, allowed us to significantly fit the estimated performances to the actual performances (\( P < 0.001 \) in each subject). The two-component model also provided a significant fit in each subject (\( P < 0.001 \)). However, the change of the RSS due to the addition of one component was significant in only two subjects (\( P < 0.05 \) in S5 and \( P < 0.01 \) in S8). A three- or four-component model did not significantly improve the fit in any subject. Table 2 shows the indicators of goodness-of-fit for each subject. Figures 1 and 2 show the fit of performances in, respectively, subjects S1 (one-component systems) and S5 (two-component systems).

The model parameter estimates were 0.046 ± 0.015 KU for the magnitude factor and 37 ± 8 days for the decay time constant with the one-component model. In the two-component model, one of the components exhibited parameters close to those obtained in the one-component model. The obtained second component exhibited a shorter decay time constant (1.9 ± 1.5 day). Table 3 shows the estimates of the model parameters in each subject for the one- and two-component models. The analysis of variance on the change in the RSS disclosed a quick component significant in only two of the subjects, S5 (\( P < 0.05 \)) and S8 (\( P < 0.01 \)). Figure 3 shows the time impulse response of the double-component systems obtained in subject S5.

**DISCUSSION**

The purpose of this study was to verify the statistical adequacy of a systems model for the description of the training effects on performance. In the previous applications of a systems model, the performance fit was not statistically tested (1, 3) or was only tested from linear regression analysis (2). The only justification of the model was a study related to the comparison between modeled responses and hormonal alterations in elite weight lifters (2). The systems model was intended to relate training to hormonal alterations, but the biological meaning of the modeled responses remained to be clarified. Moreover, the confidence of the model parameters was not tested, especially the initial status of fitness and fatigue that distorted the beginning of the studied period (2).

The statistical analysis applied in the present study shows that the systems model can significantly relate the change in performance to the training loads. However, the systems structures previously proposed, two antagonistic components (1, 2) or three components (3), are not suitable in each subject.

A one-component systems model allowed us to significantly relate the performance changes to the successive training loads in each subject at \( P < 0.001 \). The obtained function, positive in each subject with a time constant ranging from 30 to 50 days, accounts for the fitness improvement. Calvert et al. (3) have estimated the same values for endurance activities from the data of the literature. The time constants obtained by modeling were 50 days for a swimmer (3) and 45 days for long-distance runners (1).

The introduction of a second function in the performance estimation altered only weakly the function obtained in the one-component model. This further component had a time constant ranging between 1 and 5 days in the set of subjects studied. The magnitude factor was negative in six subjects. In every subject, the absolute
value of $k_3$ was higher than $k_1$. When $k_3$ is negative, a training load will thus have a negative effect on the performance during some days following the training completion (Fig. 3).

The two-component model allowed a significant fit of the performance, but the residual variations decreased significantly in only two subjects. A negative function was warranted in these two subjects only. The extension of the model to three or four components did not allow a better fit of the performances in any subject.

The RSS was probably too large to evidence a negative component in the whole group of subjects. The correlation coefficients between the estimated and actual performances show that the one-component model could take into account 61–87% of the total variations of the performances. The SE of the estimated performance ranged from 3.6 to 5.9 PU, whereas for the actual performances, it varied from 97 to 152 PU. This low precision could be due to external interferences. For instance, the model disregards the stress due to daily routine or professional or student life. This stress can influence the level of performance at every moment and thus distort the relationships between training and performance.

The precision of systems data estimation could also impair the performance fit. PS1 could underestimate the actual endurance capacity. However, it allowed us to compare the subjects with themselves and to provide a sufficient number of performance measurements during the experiment (between 32 and 36). A more reliable method of performance evaluation, as a competitive exercise, would be to investigate the responses to endurance training. This method must respect the high frequency of performance evaluation necessary to the modeling purpose, which makes it difficult to apply.

The time constants for the negative functions were 5 and 15 days in the application of Calvert et al. (3). In the present experiment, the negative effects seem to have a quicker dynamic. The time to peak performance after the completion of a training session was 9 and 10 days for the two subjects in whom a negative function was evidenced. A training load takes place in the performance estimation only 1 day after its completion; thus in a one-component model the performance peaks 1 day after the completion of training. The frequency of the performance measurements (4 times/wk and 2 days running) should have been sufficient to take into account effects that would have dynamics over several days. However, the subjects were mostly nonactive at the beginning of

The precision of systems data estimation could also
the present experiment, and the training exercise demands were rather moderate. Therefore the physiological processes involved in the present training could have been too quick (fitness improvement) or too low (fatigue) to allow us to evidence at least one negative function in each subject. Moreover, the residual variations might have masked the quick responses.

The model proposed by Calvert et al. (3) included two negative components, singling out the fatigue response from the delay in the positive response. The two-component model summarizes these two phenomena with a single function. This systems model provides a better representation of the supercompensation occurring during the recovery period (6), whereas the three-component model supposes that the gain in fitness is entirely independent of the fatigue response. The pattern of the impulse time response of the performance modeled with a two-component system (Fig. 3) includes a decrease in performance during the day following the completion of the training, then an increase up to a maximal value, followed by a decrease down to its initial level. The initial decrease of performance may be interpreted as a manifestation of fatigue. The increase up to a level higher than the initial one would thus result from both the recovery and supercompensation processes. With a two-component representation, the fitness improvement can be considered as delayed from the completion of the training, without the need for a third component. The fitness and fatigue levels could thus be estimated from the combination of the first-order functions obtained by the model computation. Moreover, the statistical analysis does not allow a biological meaning to be ascribed to each obtained function.

The model parameter estimated for the presently studied subjects cannot be expected to express the responses to a more strenuous training. Stronger and more frequent exercise demands would induce a greater fatiguing effect. The present formulation of the systems model does not alter the time durations to recovery and to peak performance, whatever the level of training input. Moreover, the response to a training load at any later given time is independent of the events interposed between the training session and this target time. The linear time-invariant first-order systems can thus be unsuitable for the representation of the training responses, especially in athletes whose training is composed of a succession of periods of intensive and reduced training. A better formulation of the model should be investigated using data obtained from elite athletes or from an experimental program, including periods of intensive training. Internal parameters of the system (e.g., biological indexes of fatigue or fitness) would be helpful in the interpretation of the system's behavior and in the improvement of the model formulation.

In conclusion, the present study shows that a systems model can significantly relate the performance changes to the successive training loads. A systems model composed of two antagonistic first-order transfer functions is supposed to provide a proper representation of the training responses. However, the low level of training demands and the inaccuracy of the fitting could have impaired the evidence of a fatigue effect during the present training. Further investigations with data related to more strenuous and more varied training regimens are needed to complete the analysis of the adequacy of the systems model and to improve its formulation.

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